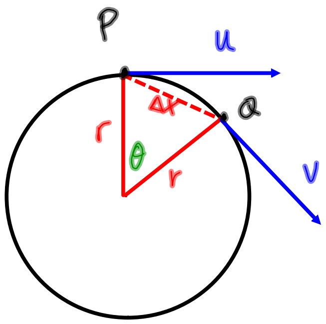
2.4 Uniform circular motion

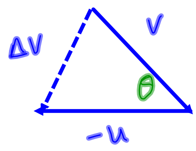
- uniform -> constant speed on a circular path
- the velocity continuously changes since its direction changes.
- since there is a change in velocity, there is acceleration.
- acceleration is the rate of change of velocity

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

- the object is continuously accelerating since the velocity is continuously changing

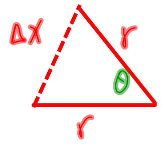
Direction of Acceleration





$|v| = |u| = v$

(uniform motion)



as $\Delta t \to 0$, $\theta \to 0$

Also, the distance travelled along the circular path approaches Δx

Note: $u \perp r$, $v \perp r$ } tangent to curve

$$\frac{\Delta v}{v} = \frac{\Delta x}{r}$$

$$\Delta v = \frac{\Delta x\, v}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta x}{\Delta t}\,\frac{v}{r}$$

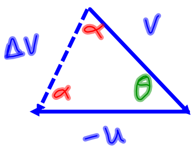
approaches the actual distance when $\Delta t \to 0$

$$\frac{\Delta v}{\Delta t} = v\left(\frac{v}{r}\right)$$

$$a_c = \frac{v^2}{r}$$

Centripetal acceleration

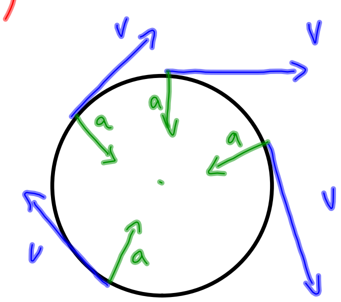
"centreseeking"



As $\Delta t \to 0$, $\theta \to 0$

which means that $\alpha \to 90^\circ$

So $\Delta v \perp v$, the acceleration is in the same direction as Δv and will be $\perp$ to v (i.e. along the radius of curvature)



Period and frequency for circular motion

Period T → time for one rotation/revolution/vibration.

→ unit: s $\frac{\text{time}}{\text{rotations}}$

frequency f → how many rotations/revolutions/vibrations in a given time (usually 1 s) $\frac{\text{rotations}}{\text{time}}$

→ s^{-1} or Hz (hertz)

Note that period and frequency are reciprocals of one another.

$$T = \frac{1}{f} \quad \underline{\text{or}} \quad f = \frac{1}{T}$$

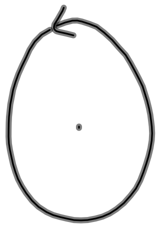
Alternative Expressions for Centripetal Acceleration

* $a_c = \frac{v^2}{r}$

for 1 complete revolution around a circle:

Data booklet

$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$



$v = \frac{\Delta x}{\Delta t}$

$v = \frac{2\pi r}{T}$

(tangential speed)

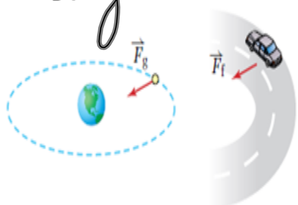
* $a_c = \frac{4\pi^2 r}{T^2}$

Not in data booklet. $a_c = 4\pi^2 r f^2$

Centripetal Force

FBDs are <u>VERY</u> important when solving centripetal force problems!!!

Since a body travelling a circular path is continuously accelerating, there must be a net force (unbalanced force) acting on the body.



F_c is the the net force or unbalanced force!
↑ centripetal.

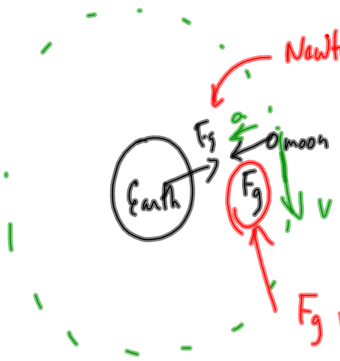
$$\vec{F}_{net} = m\vec{a}$$

$$F_c = \frac{mv^2}{r}$$

F_{net}

(there may be more than one force that results in F_{net} (F_c))

Consider the moon orbiting the Earth:



Newton's 3rd law -> the moon exerts a force on the Earth equal to the force that the Earth exerts on the moon but in the opposite direction.

F_g is the centripetal force on the moon ← holds the moon in its orbit

it is directed to the centre of the Earth (in the same direction of the acceleration)

Example:
The speed of the Earth in its orbit about the Sun is about 3.0 x 10^4 m s^{-1} and the radius of its orbit is 1.5 x 10^{11} m. Calculate the centripetal acceleration of the Earth.

$$a_c = \frac{v^2}{r}$$ ← magnitude

$$a_c = \frac{(3.0 \times 10^4 \text{ m s}^{-1})^2}{1.5 \times 10^{11} \text{ m}}$$

$$a_c = 6.0 \times 10^{-3} \text{ m s}^{-2}$$ ← direction: towards the Sun.

Example:

The period or rotation of a bicycle wheel is 0.25 s. What is its rotation frequency?

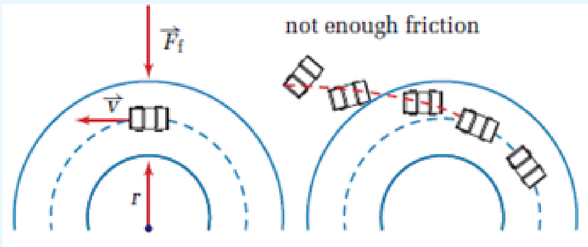
$$f = \frac{1}{T}$$

$$f = \frac{1}{0.25\text{s}}$$

$$f = 4.0 \text{ Hz}$$

Centripetal Force in a Horizontal and a Vertical Plane

1. A car with a mass of 2135 kg is rounding a curve on a level road. If the radius of curvature of the road is 52 m and the coefficient of friction between the tires and the road is 0.70, what is the maximum speed at which the car can make the curve without skidding off the road?



continued ▶

2. You are playing with a yo-yo with a mass of 225 g. The full length of the string is 1.2 m. You decide to see how slowly you can swing it in a vertical circle while keeping the string fully extended, even when the yo-yo is at the top of its swing.
   (a) Calculate the minimum speed at which you can swing the yo-yo while keeping it on a circular path.
   (b) At the speed that you determine in part (a), find the tension in the string when the yo-yo is at the side and at the bottom of its swing.

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